Financial Frictions with Risk, Irreversible Capital, and Default

Francisco Buera[1,2] Jeremy Majerovitz[1,2] Yongseok Shin[1,2]
Kuldeep Singh[1]

[1]Washington University in St. Louis

[2]Federal Reserve Bank of St. Louis

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Motivation

- Large cross-country income per-capita differences
- Credit frictions play a role
- Many models of credit frictions ignore investment risk, irreversibility
- Micro-development evidence: limited insurance $>$ limited credit?
  - Udry (SED, 2012); Karlan, Osei, Osei-Akoto and Udry (2014)

Our Questions

- Quantitative model of risk-averse entrepreneurs, irreversible investment, limited commitment/default
- Study how development outcomes are affected by the contractual environment
  - Relationship between economic and financial development?
  - Role of resalability frictions, collateral requirements?
  - Does a default option discourage/promote development?
  - Poverty trap?

Related Literature

- Investment risk and capital accumulation: Angeletos (2007), Sandri (2014)
- Irreversible investment and misallocation: Asker, Collard-Wexler, De Loecker (2014); Boar, Gorea, Midrigan (2023)
- Entrepreneurial risk and default: Akyol and Athreya (2011); Morales (2022)
- Endogenous entrepreneurial risk: Vereshchagina and Hopenhayn (2009); Robinson (2023)

Model Overview: Risk and Irreversibility

Start with entrepreneurs who invest under financial frictions

Standard model features strong "save-your-way-out" dynamics

- Productivity is known in a given period
- Rental market for capital subject to collateral constraint
- $\implies$ No investment risk
- $\implies$ Invest all the way up to the constraint, until $r + \delta = f'(k)$

Here: risk-averse entrepreneurs make partially irreversible investments in capital, subject to the risk that their productivity may change in the future.

Entrepreneurs invest less in the firm because they are risk averse, so capital accumulation is slow.

(Under CRRA, entrepreneur eventually gets so rich that she is no longer risk averse in CARA sense.)

Model Overview: Default

How does credit enter the picture?

Credit interacts with risk if the entrepreneur can default.

The option to default creates a state-contingent contract: if things get very bad, entrepreneur can default and not pay off debts.

Makes investment less risky. (Theoretically, may even overturn the underinvestment due to risk.)

Punishment for default is loss of capital and access to credit (temporary).

Zero profits for banks $\implies$ default raises borrowers' interest rate

Entrepreneur's Problem

$$\max_{c_t, i_t, \tau} \ \mathbb{E}_0 \int_0^{\tau} e^{-\rho t} \frac{c_t^{1-\sigma}}{1-\sigma} dt + e^{-\rho \tau} V^{def}(0,0,z_t)$$

$$da = \big(\pi(k_t, z_t) + w + r_a a_t - c_t - i_t - \Phi(i_t, k_t)\big) dt$$

$$dk = (i_t - \delta k_t) dt$$

$$a_t \geq -\lambda k_t, \quad k_t \geq 0$$

$$\pi(k_t, z_t) \equiv \max_{l} z_t k_t^{\alpha} l_t^{\beta} - wl$$

$$z_t \in \{z_1, z_2, z_3\}$$

- Borrow ($a \leq 0$) at rate $r_a = r_b$.
- Save ($a > 0$), at rate $r_a = r_s$ ($r_s \leq r_b$).
- Stochastic productivity transitions (Poisson)
- Can choose to default (optimal stopping time τ); gets $V^{def}(0,0,z_t)$ (lose capital and credit access)

Entrepreneur's Problem in Default

$$\max_{c_t, i_t} \mathbb{E}_0 \int_0^T e^{-\rho t} \frac{c_t^{1-\sigma}}{1-\sigma} dt + e^{-\rho T} V(a_T, k_T, z_T)$$

$$da = \big(\pi(k_t, z_t) + w + r_s a_t - c_t - i_t - \Phi(i_t, k_t)\big) dt$$

$$dk = (i_t - \delta k_t) dt$$

$$a_t \geq 0, \quad k_t \geq 0$$

When the entrepreneur defaults:

- Debt goes to zero, but the bank takes all capital
- Cannot borrow until...
- Regain credit access with Poisson intensity χ_{dn}.

Adjustment Costs and Partial Irreversibility

When the firm invests, needs to pay adjustment cost, $\Phi(i,k)$, in addition to cost of investment.

When investment is negative (selling off capital), only gets back $\phi \leq 1$ dollars for each dollar of capital sold.

Also pays small quadratic adjustment cost: this is just to make the problem smooth

$$\Phi(i,k) = \begin{cases} \frac{\kappa}{2}\left[\frac{i}{k+\bar{k}}\right]^2 (k+\bar{k}) & i \geq 0 \\ \\ -(1-\phi)i + \frac{\kappa}{2}\left[\frac{i}{k+\bar{k}}\right]^2 (k+\bar{k}) & i < 0 \end{cases}$$

Value Functions

With credit access:

$$\rho V(a,k,z_j) = \max\Bigg\{\rho V^{def}(0,0,z_j),$$

$$\max_{c,i}\Bigg[\frac{c^{1-\sigma}}{1-\sigma} + V_a\cdot\Big(\pi(k,z_j) + w + r_a\cdot a - c - i - \Phi(i,k)\Big)$$

$$+ V_k\cdot(i-\delta k) + \sum_{-j}\lambda_{z_{j,-j}}\big(V(a,k,z_{-j}) - V(a,k,z_j)\big)\Bigg]\Bigg\}$$

Without credit access:

$$\rho V^{def}(a,k,z_j) =$$

$$\max_{c,i}\Bigg[\frac{c^{1-\sigma}}{1-\sigma} + V_a^{def}\cdot\Big(\pi(k,z_j) + w + r_a\cdot a - c - i - \Phi(i,k)\Big)$$

$$+ V_k^{def}\cdot(i-\delta k) + \sum_{-j}\lambda_{z_{j,-j}}\big(V^{def}(a,k,z_{-j}) - V^{def}(a,k,z_j)\big)$$

$$+ \chi_{dn}\cdot\big(V(a,k,z_j) - V^{def}(a,k,z_j)\big)\Bigg]$$

Bank's Problem

Bank lends at r_b with loan-to-value constraint (requires $1/\lambda$ units of capital as collateral for each dollar of debt)

Same r_b for all borrowers: does not depend on (a, k, z). May result from information constraints and/or legal constraints.

If entrepreneur defaults, bank liquidates the firm and gets back $\phi_b \cdot k$. In the baseline model, bank has same liquidation technology as entrepreneur $(\phi_b = \phi)$.

Bank borrows at rate r_s, perfectly elastic supply. Makes zero profit in equilibrium.

Bank's Zero Profit Condition

Let $G(a,k,z)$ be the joint c.d.f.

Δ is a small time interval.

$\mathcal{I}_{def}$ is the default region (changes with Δ)

Let $B := \int_{a\leq 0} -a dG(a,k,z)$ denote total debt.

Zero profits implies (discrete time approximation):

$$r_s B\Delta = r_b B\Delta + \int_{(a,k,z)\in\mathcal{I}_{def}} (\phi_b \cdot k - (-a)) dG(a,k,z)$$

Limit as $\Delta \to 0$ (continuous time):

$$r_b = r_s + \text{Default Risk Premium}$$

$$\text{Default Risk Premium} = \lim_{\Delta\to 0} \frac{\int_{(a,k,z)\in\mathcal{I}_{def}} (\phi_b \cdot k + a) dG(a,k,z)}{B\Delta}$$

Planner's Problem (Static)

To measure deviations from efficiency, solve planner's problem.

Given distribution $\Omega(k, z)$, allocation of labor is a static problem:

$$\max_{l} \int zk^{\alpha} l^{\beta} d\Omega \quad s.t \quad \int l d\Omega = 1$$

$$l(k, z) = \frac{\left(zk^{\alpha}\right)^{\frac{1}{1-\beta}}}{\int \left(zk^{\alpha}\right)^{\frac{1}{1-\beta}} d\Omega}$$

However, planner must solve dynamic investment/liquidation problem...

Planner's Problem (Dynamic)

Planner takes into account NPV of resource flows:

$$\max_{i_t} \int_0^\infty e^{-\rho t} \int \Big(y(k_t, z_t) - i_t - \Phi(i_t, k_t) - MPL_t \cdot l(k_t, z_t) \Big) d\Omega_t dt$$

$$dk = (i_t - \delta k_t) dt$$

where

$$y(k_t, z_t) = \frac{z_t^{\frac{1}{1-\beta}} k_t^{\frac{\alpha}{1-\beta}}}{\left[\int \left(z k^\alpha \right)^{\frac{1}{1-\beta}} d\Omega \right]^\beta}$$

Planner invests less than static optimum: takes into account costly liquidation due to negative productivity shocks.

Parameters for Numerical Example

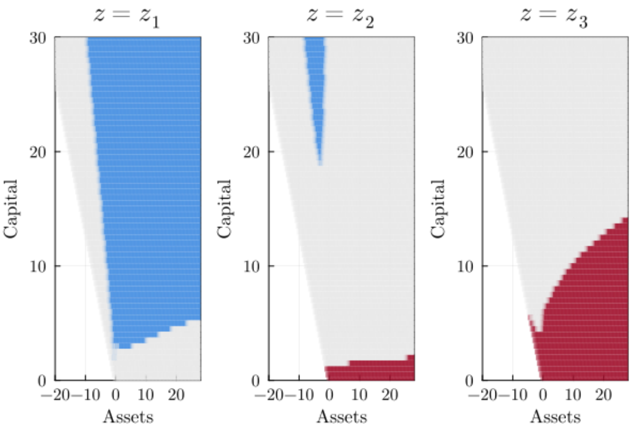
Parameters	Values
ρ (discount rate)	0.083
σ (risk aversion)	2
r_s (saving rate)	0.02
δ (depreciation)	0.06
$\phi, \kappa, \bar{k}$ (adjustment cost)	0.35,0.1,0.1
α, β (production function)	0.3,0.49
χ_{dn} (regain credit access)	0.5
z_1, z_2, z_3 (productivity)	0,1.45,1.75
λ (loan-to-value constraint)	0.75

Table: Productivity Process

	z_1	z_2	z_3	Stationary Mass
z_1	-	0.05	0.02	0.87
z_2	1	-	0.1	0.05
z_3	0.125	0.15	-	0.08

Decision Rules

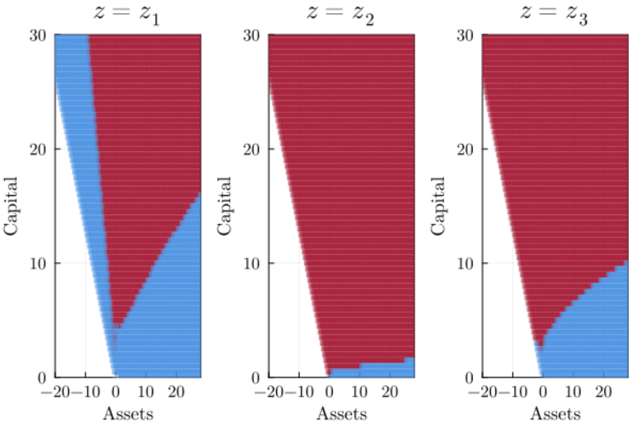
Figure: Investment



Red: $i > 0$ Gray: $i = 0$ Blue: $i < 0$

Decision Rules

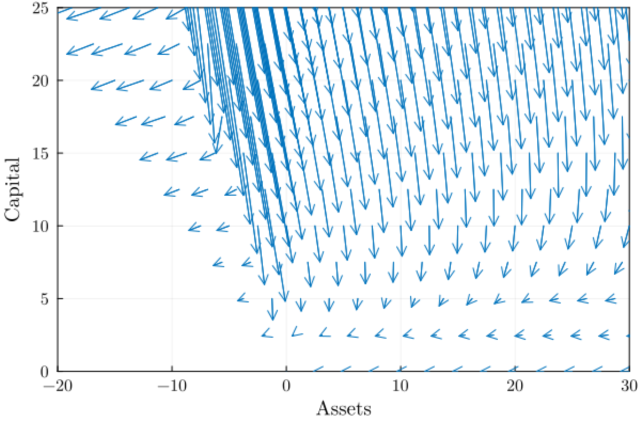
Figure: Saving



Red: $\dot{a} > 0$ Gray: $\dot{a} = 0$ Blue: $\dot{a} < 0$

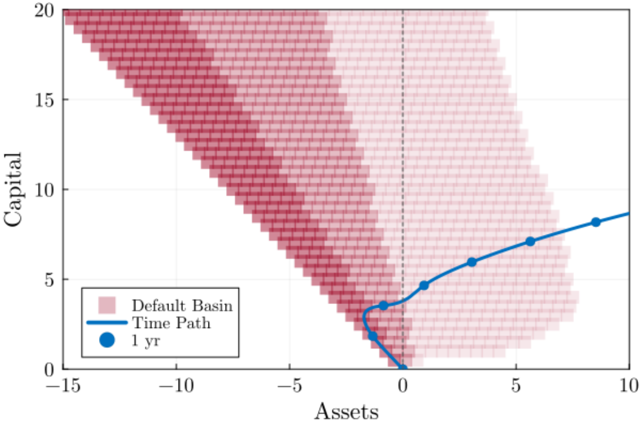
Phase Diagram

Figure: $z = z_1$



Time Path and Default Basin

Figure: $z = z_3$ starting with $k = 0$, $a = 0$



"Default Basin" for $z = z_1$: 1, 5, 25 yrs

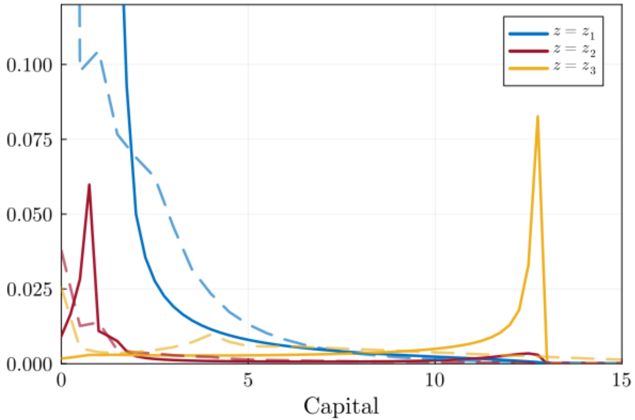
Role of Default Option

Variable	No Default	With Default
$r_b(borrowingrate)$	2%	26.4%
Fraction without credit access	0	0.031
Default rate	-	0.41
$a < 0$	0.240	0.037
$Y/Y^p_{\phi=0.35}$	0.85	0.826
$K/K^p_{\phi=0.35}$	0.858	0.743
$K_{z_1}/K^p_{z_1,\phi=0.35}$	1.107	0.886
$K_{z_2}/K^p_{z_2,\phi=0.35}$	0.740	0.688
$K_{z_3}/K^p_{z_3,\phi=0.35}$	0.616	0.604
$TFP/TFP^p_{\phi=0.35}$	0.9	0.924

- Less investment with default option in the long run
- Lower output in the long run but moderately better allocative efficiency (less capital by z_1)

Planner vs. Entrepreneur with Default Option

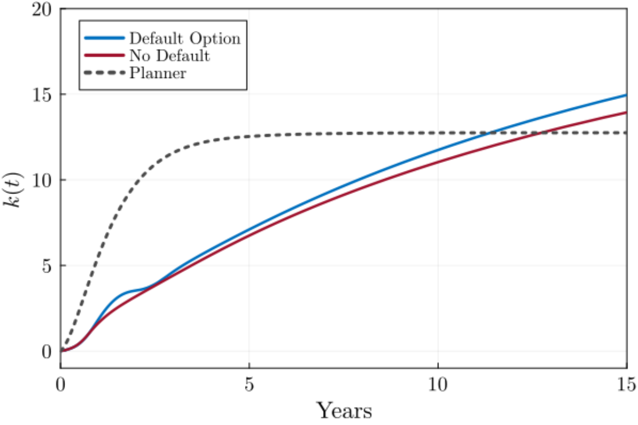
Figure: Stationary Density of k by z



Solid lines: Planner; Dashed lines: Benchmark with default option

Planner vs. Entrepreneur: Capital Paths

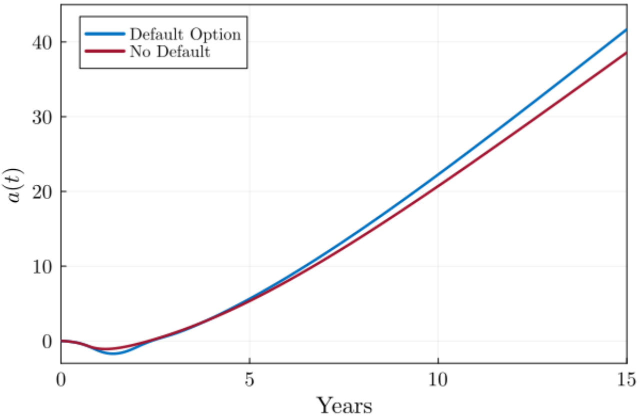
Figure: $z = z_3$ starting with $a = 0$, $k = 0$



- Because of risk, entrepreneurs accumulate more slowly than planner
- Faster capital growth due to leverage (early on) and default option

Entrepreneur: Asset Paths

Figure: $z = z_3$ starting with $a = 0$, $k = 0$



Comparative Statics w.r.t. ϕ

Variable	$\phi = 0.1$	$\phi = 0.27$	$\phi = 0.35$	$\phi = 0.75$
r_b	110%	42.8%	26.4%	2%
Fraction without credit access	0.027	0.03	0.031	0.006
Default Rate	0.78	0.47	0.41	0.021
$a < 0$	0.018	0.032	0.037	0.15
$Y/Y^p_{\phi=0.35}$	0.775	0.806	0.826	1.007
$K/K^p_{\phi=0.35}$	0.867	0.857	0.743	0.891
$K_{z_1}/K^p_{z_1,\phi=0.35}$	1.25	1.174	0.886	0.519
$K_{z_2}/K^p_{z_2,\phi=0.35}$	0.691	0.712	0.688	1.115
$K_{z_3}/K^p_{z_3,\phi=0.35}$	0.5	0.548	0.604	1.235
$TFP/TFP^p_{\phi=0.35}$	0.818	0.855	0.924	1.052
$K^p/K^p_{\phi=0.35}$	1.755	1.271	1	0.914

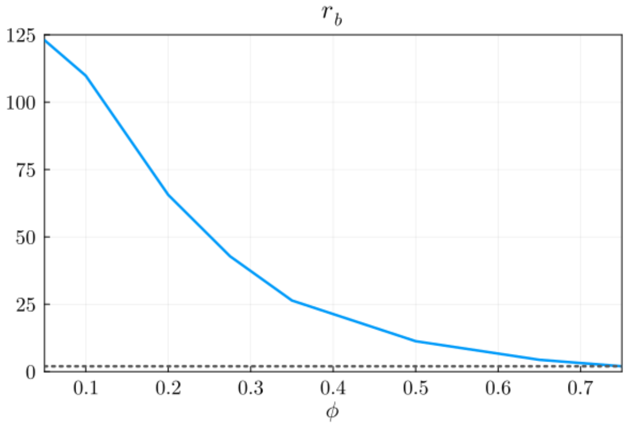
ϕ: fraction remains after sale of capital (ϕ_b for bank)

Less friction (higher ϕ) leads to in the long run

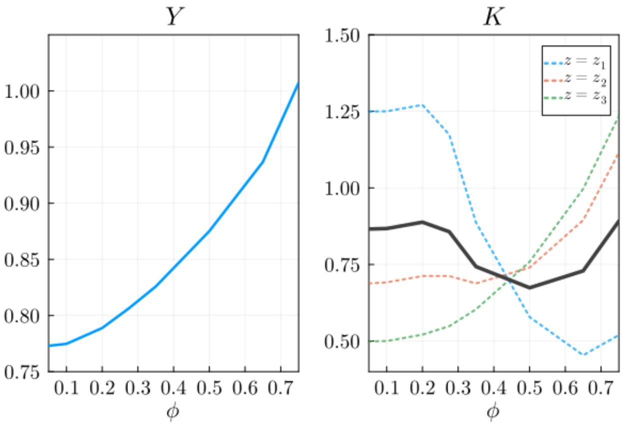
- More borrowing, less default
- More output, more investment by z_3
- Better allocative efficiency

Comparative Statics w.r.t. ϕ

Figure: Borrowing Rate (%)



Comparative Statics w.r.t. ϕ

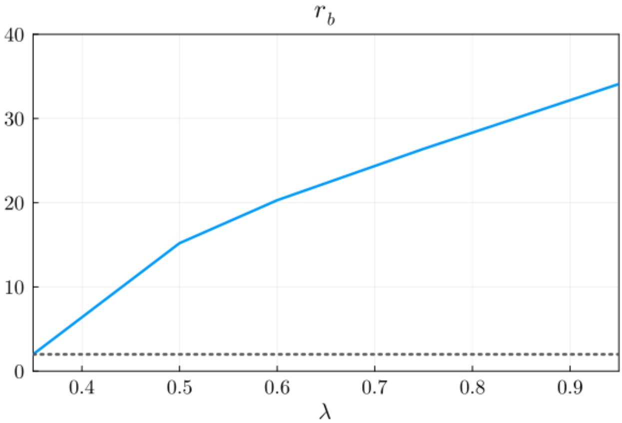


All values normalized by the corresponding planner value with $\phi = 0.35$

- ϕ captures... technology, contractual frictions (e.g., asymmetric information), market thickness

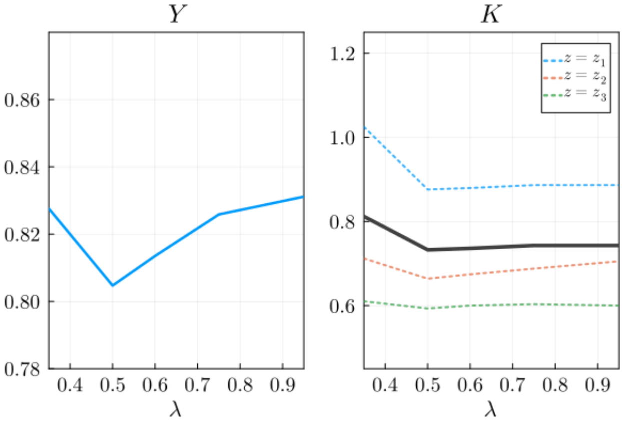
Comparative Statics w.r.t. λ

Figure: Borrowing Rate (%)



λ: loan-to-value constraint

Comparative Statics w.r.t. λ



- λ affects borrowing, but has muted effects on quantities (long run).

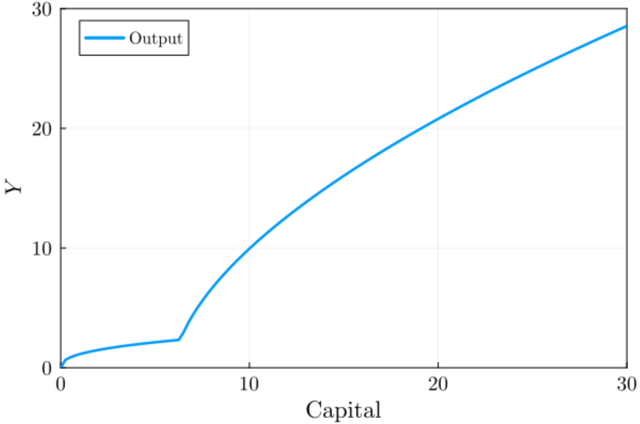
Unpacking Comparative Statics w.r.t. ϕ and ϕ_b

Variable	$\phi_b = 0.35$	$\phi_b = 0.35$	$\phi_b = 0.65$	$\phi_b = 0.65$
ϕ	0.65	0.35	0.35	0.65
r_b	13.3%	26.4%	5.5%	4.4%
Without credit access	0.027	0.031	0.034	0.028
Default Rate	0.34	0.41	0.26	0.29
$a < 0$	0.038	0.037	0.065	0.048
$Y/Y^p_{\phi=0.35}$	0.918	0.826	0.86	0.936
$K/K^p_{\phi=0.35}$	0.693	0.743	0.86	0.73
$K_{z_1}/K^p_{z_1,\phi=0.35}$	0.435	0.886	1.047	0.453
$K_{z_2}/K^p_{z_2,\phi=0.35}$	0.85	0.688	0.79	0.894
$K_{z_3}/K^p_{z_3,\phi=0.35}$	0.934	0.604	0.676	0.997
$TFP/TFP^p_{\phi=0.35}$	1.055	0.924	0.911	1.055

ϕ: fraction remains after sale of capital (ϕ_b for bank)

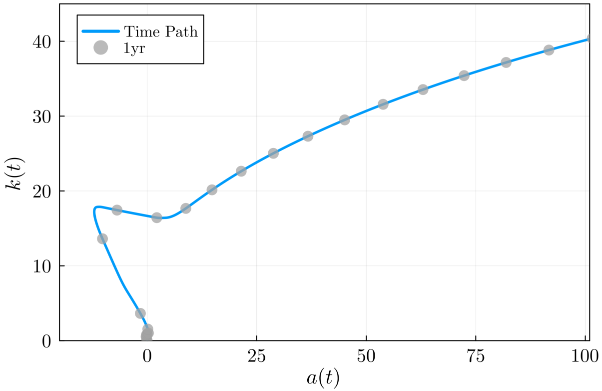
- ϕ has larger effects on quantities, but ϕ_b has larger effects on borrowing and default (long run).

Role of Default Option: Skiba (1978) Technology



Skiba Technology: Time Paths

Figure: $z = z_3$ starting with $a = 0$, $k = 0$



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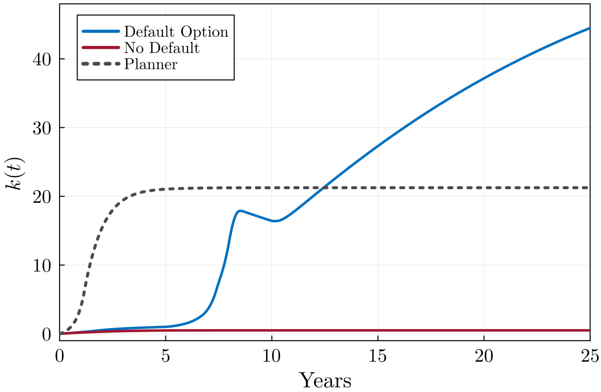
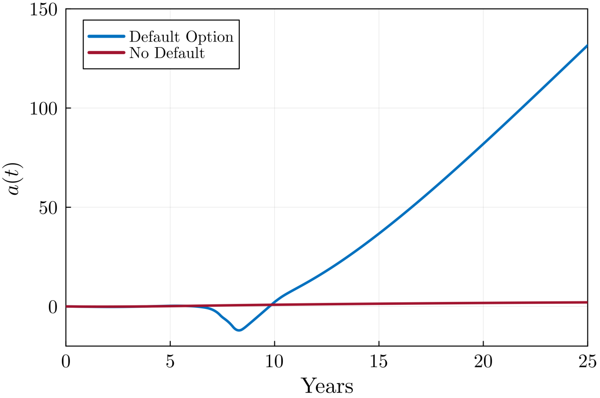


- Unproductive at small scale, slower investment, followed by big jump financed by (defaultable) debt
- Without a default option, poverty trap

Skiba Technology: Time Paths

Figure: $z = z_3$ starting with $a = 0$, $k = 0$



Taking Stock

- Uninsurable investment risk due to irreversibility can lead to significant underinvestment (more so than collateral constraint)
- The option to default can be an important insurance mechanism overcoming this (partially), especially with non-convex production functions (e.g., high fixed cost)
- Work in progress: Differentiated loan contracts, richer/better quantification strategy...